The House of Representatives Apportionment Formula: An Analysis of Proposals for Change and Their Impact on States

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August 6, 2013

Congressional Research Service
7-....
www.crs.gov
R41382

Summary

As a basis for understanding the reallocation of Representatives among the states based on the 2010 Census, it may prove helpful to examine the current House of Representatives apportionment formula. In addition, some members of the statistical community have, in the recent past, urged Congress to consider changing the current apportionment formula. Consequently, an examination of other methods that could be used to apportion the seats in the House of Representatives may contribute to a deeper understanding of the apportionment process.

Seats in the House of Representatives are allocated by a formula known as the method of equal proportions or the "Hill" method. If Congress decided to change it, there are at least five alternatives it might consider. Four of these are based on rounding fractions and one, on ranking fractions. The current apportionment system (codified in 2 U.S.C. 2a) also is based on rounding fractions.

The Hamilton-Vinton method, used to apportion the House of Representatives from 1851-1901, is based on ranking fractions. First, the total population of the 50 states is divided by 435 (the House size) in order to find the national "ideal sized" district. Next, this number is divided into each state's population, producing the state's "quota" of seats. Each state is then awarded the whole number in its quota (but at least one). If fewer than 435 seats have been assigned by this process, the fractional remainders of the 50 states are rank-ordered from largest to smallest, and seats are assigned in this manner until 435 are allocated.

The rounding methods, including the Hill method currently in use, allocate seats among the states differently, but operationally the methods only differ by where rounding occurs in seat assignments. Three of these methods—Adams, Webster, and Jefferson—have fixed rounding points. Two others—Dean and Hill—use varying rounding points that rise as the number of seats assigned to a state grows larger. The methods can be defined in the same way (after substituting the appropriate rounding principle in parentheses). The rounding point for Adams is (up for all fractions); for Dean (at the harmonic mean); for Hill (at the geometric mean); for Webster (at the arithmetic mean, which is 0.5 for successive numbers); and for Jefferson (down for all fractions). Substitute these phrases in the general definition below for the rounding methods:

> *Find a number so that when it is divided into each state's population and resulting quotients are rounded (substitute appropriate phrase), the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional requirement.)*

Fundamental to choosing an apportionment method is a determination of fairness. Each apportionment method discussed in this report has a "rational" basis, and for each, there is at least one test according to which it is the most equitable. The question of how the concept of fairness can best be defined, in the context of evaluating an apportionment formula, remains open. Which of the mathematical tests discussed in this report best approximates the constitutional requirement that Representatives be apportioned among the states according to their respective numbers is, arguably, a matter of judgment, rather than an indisputable mathematical test.

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The House of Representatives Apportionment Formula: An Analysis of Proposals for Change and Their Impact on States[1]

Introduction

As a basis for understanding the reallocation of Representatives among the states based on the 2010 Census, it may prove helpful to examine the current House of Representatives apportionment formula. In addition, some members of the statistical community have, in the recent past, urged Congress to consider changing the current apportionment formula.[2] Consequently, an examination of other methods that could be used to apportion the seats in the House of Representatives may contribute to a deeper understanding of the apportionment process.

In 1991, the reapportionment of the House of Representatives was nearly overturned because the current "equal proportions" formula[3] for the House apportionment was held to be unconstitutional by a three-judge panel of a federal district court. The court concluded that,

> By complacently relying, for over fifty years, on an apportionment method which does not even consider absolute population variances between districts, Congress has ignored the goal of equal representation for equal numbers of people. The court finds that unjustified and avoidable population differences between districts exist under the present apportionment, and ... [declares] section 2a of Title 2, United States Code unconstitutional and void.[4]

The three-judge panel's decision came almost on the 50th anniversary of the current formula's enactment.[5]

The government appealed the panel's decision to the Supreme Court, where Montana argued that the equal proportions formula violated the Constitution because it "does not achieve the greatest possible equality in number of individuals per Representative." This reasoning did not prevail, because, as Justice Stevens wrote in his opinion for a unanimous court, absolute and relative differences in district sizes are identical when considering deviations in district populations *within* states, but they are different when comparing district populations *among* states. Justice Stevens noted, however, that "although common sense" supports a test requiring a "good faith effort to achieve precise mathematical equality *within* each State ... the constraints imposed by Article I,

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[1] A similar, previous CRS report was authored by (name redacted), who retired from CRS in 2005. While the current report is modified by the author, Mr. Huckabee's contributions, in large part, remain. Of course, any errors that may appear are due solely to the current author.

[2] See Brookings Institution Policy Brief, *Dividing the House: Why Congress Should Reinstate the Old Reapportionment Formula*, by H. Peyton Young, Policy Brief No. 88, (Washington, Brookings Institution, August 2001). Young suggests that Congress consider the matter "now—well in advance of the next census," p. 1

[3] CRS Report R41357, *The U.S. House of Representatives Apportionment Formula in Theory and Practice*, by (name redacted).

[4] *Montana v. Department of Commerce*, No. CV. 91-22-H-CCL (D. Mt. October 18, 1991). U.S. District Court for the District of Montana, Helena Division.

[5] 55 *Stat.* 761, codified in 2 U.S.C. 2a, was enacted November 15, 1941.

§2, itself make that goal illusory for the nation as a whole." He concluded "that Congress had ample power to enact the statutory procedure in 1941 and to apply the method of equal proportions after the 1990 census."[6]

The year 1991 was a banner year for court challenges to the apportionment process. At the same time the Montana case was being argued, another case was being litigated by Massachusetts. The Bay State lost a seat to Washington because of the inclusion of 978,819 federal employees stationed overseas in the state populations used to determine reapportionment. The court ruled that Massachusetts could not challenge the President's decision to include the overseas federal employees in the apportionment counts, in part because the President is not subject to the terms of the Administrative Procedure Act.[7]

In 2001, the Census Bureau's decision to again include the overseas federal employees in the population used to reapportion the House produced a new challenge to the apportionment population. Utah argued that it lost a congressional seat to North Carolina because of the Bureau's decision to include overseas federal employees in the apportionment count, but not other citizens living abroad. Utah said that Mormon missionaries were absent from the state because they were on assignment: a status similar to federal employees stationed overseas. Thus, the state argued, the Census Bureau should have included the missionaries in Utah's apportionment count. The state further argued that, unlike other U.S. citizens living overseas, missionaries could be accurately reallocated to their home states because the Mormon Church has excellent administrative records. Utah's complaint was dismissed by a three-judge federal court on April 17, 2001.[8]

The Supreme Court appears to have settled the issue about Congress's discretion to choose a method to apportion the House, and has granted broad discretion to the President in determining who should be included in the population used to allocate seats.

What, if any, challenges to the apportionment formula and process the country will face after the 2010 Census and apportionment remain to be seen. Although modern Congresses have rarely considered the issue of the formula used in the calculations, this report describes apportionment options from which Congress could choose and the criteria that each method satisfies.[9]

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[6] *Department of Commerce v. Montana* 503 U.S. 442 (1992).

[7] *Franklin* v. *Massachusetts*, 505 U.S. 788 (1992). The Administrative Procedure Act (APA)(5 U.S.C, Subchaper II) sets forth the procedures by which federal agencies are accountable to the public and their actions are subject to review by the courts. Since the Supreme Court ruled that a President's decisions are not subject to review under the APA by courts, the district court's decision to the contrary was reversed. Plaintiffs in this case also challenged the House apportionment formula, arguing that the Hill (equal proportions) method discriminated against larger states.

[8] *Utah* v. *Evans*, No. F-2-01-CV-23: B (D. Utah, complaint filed January 10, 2000). Representative Gilman introduced H.R. 1745, the Full Equality for Americans Abroad Act, on May 8, 2001. The bill would require including all citizens living abroad in the state populations used for future apportionments. For further reading on this and other legal matters pertaining to the 2000 census, see CRS Report RL30870, *Census 2000: Legal Issues re: Data for Reapportionment and Redistricting*, by (name redacted).

[9] Representative Fithian (H.R. 1990) and Senator Lugar (S. 695) introduced bills in the 97th Congress to adopt the Hamilton-Vinton method of apportionment to be effective for the 1981 apportionment and subsequent apportionments. Hearings were held in the House, but no further action was taken.

Background

One of the fundamental issues before the framers at the constitutional convention in 1787 was the allocation of representation in Congress between the smaller and larger states. The solution ultimately adopted, known as the Great (or Connecticut) Compromise, resolved the controversy by creating a bicameral Congress with states represented equally in the Senate, but in proportion to population in the House.[10]

The Constitution provided the first apportionment: 65 Representatives were allocated to the states based on the framers' estimates of how seats might be apportioned following a census.[11] House apportionments thereafter were to be based on Article 1, section 2, as modified by clause 2 of the Fourteenth Amendment:

> Amendment XIV, section 2. Representatives shall be apportioned among the several States according to their respective numbers....
>
> Article 1, section 2. The number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at least one Representative....

The constitutional mandate that Representatives would be apportioned according to population did not describe how Congress was to distribute fractional entitlements to Representatives. Clearly there would be fractions because districts could not cross state lines and the states' populations were unlikely to be evenly divisible. From its beginning in 1789, Congress was faced with questions about how to apportion the House of Representatives. The controversy continued until 1941, with the enactment of the Hill ("equal proportions") method.

During congressional debates on apportionment, the major issues were how populous a congressional district ought to be (later recast as how large the House ought to be), and how fractional entitlements to Representatives should be treated. The matter of the permanent House size has received little attention since it was last increased to 435 after the 1910 Census.[12] The Montana legal challenge added a new perspective to the picture—determining which method comes closest to meeting the goal of "one person, one vote."

The "one person, one vote" concept was established through a series of Supreme Court decisions beginning in the 1960s. The court ruled in 1962 that state legislative districts must be approximately equal in population (*Baker v. Carr*, 369 U.S. 186). This ruling was extended to the U.S. House of Representatives in 1964 (*Wesberry v. Sanders*, 376 U.S. 1). Thus far, the "one person, one vote" concept has only been applied within states. States must be able to justify any deviations from absolute numerical equality for their congressional districts in order to comply with a 1983 Supreme Court decision—*Karcher v. Daggett* (462 U.S. 725).[13]

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[10] For a discussion and analysis, see Charles A. Kromkowski, *Recreating the Republic* (Cambridge, UK: Cambridge University Press, 2002), pp. 261-307.

[11] There was even a dispute over the first apportionment, see Kromkowski, pp. 287-294.

[12] Article I, Section 3 defines both the maximum and minimum size of the House; the actual House size is set by law. There can be no less than one Representative per state, and no more than one for every 30,000 persons. Thus, the House after 2010 could be as small as 50 and as large as 10,306 Representatives (30,000 divided into the total U.S. apportionment population).

[13] CRS Report RS22479, *Congressional Redistricting: A Legal Analysis of the Supreme Court Ruling in League of United Latin American Citizens (LULAC) v. Perry*, by (name redacted).

The population distribution among states in the 2010 Census, combined with a House size of 435, and the requirement that districts not cross state lines, means that there is a wide disparity in district sizes—from 527,624 (for Rhode Island's two congressional districts) to 994,416 (for Montana's single district) after the 2010 Census. This *interstate* population disparity among districts in 2010 contrasts with the *intrastate* variation experienced in the redistricting process. Thirty of the 43 states that had two or more districts in 2012 drew districts with a population difference between their largest and smallest districts of fewer than 10 and, of these, 26 states had a difference between their largest and smallest congressional district, with respect to population, of one or less. Only three states varied by more than 1,000 persons.[14]

Given a fixed-size House and an increasing population, there will inevitably be population deviations in district sizes among states. What should be the goal of an apportionment method? Although Daniel Webster was a proponent of a particular formula (the major fractions method), he succinctly defined the apportionment problem during debate on an apportionment bill in 1832 (4 Stat. 516). Webster said that,

> The Constitution, therefore, must be understood, not as enjoining an absolute relative equality, because that would be demanding an impossibility, but as requiring of Congress to make the apportionment of Representatives among the several states according to their respective numbers, as near as may be. That which cannot be done perfectly must be done in a manner as near perfection as can be.[15]

Which apportionment method is the "manner as near perfection as can be"? Although there are potentially thousands of different ways in which the House could be apportioned, six methods are most often mentioned as possibilities. These are the methods of Hamilton-Vinton, "largest fractional remainders"; of Adams, "smallest divisors"; of Dean, "harmonic mean"; of Hill, "equal proportions"; of Webster, "major fractions"; and of Jefferson, "largest divisors."[16]

Apportionment Methods Defined

Since 1941, seats in the House of Representatives have been apportioned according to the method of equal proportions (Hill)(see below, in "Rounding Methods").[17] However, from 1790 to the present, alternative methods for apportioning seats have been used or seriously considered. Six such methods stand out. One, the Hamilton-Vinton method, involves ranking fractional remainders. The others (the methods of Adams, Dean, Hill, Webster, and Jefferson) involve rounding fractional remainders.

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[14] The 30 states that had population differences between their most populous congressional district and their least populous congressional district of fewer than 10 persons were Alabama, Arizona, California, Colorado, Connecticut, Florida, Georgia, Illinois, Indiana, Maine, Massachusetts, Michigan, Minnesota, Missouri, Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Carolina, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, South Carolina, Tennessee, Utah, and Wisconsin. The three states with a difference greater than 1,000 persons were Maryland, Virginia, and West Virginia.

[15] M. L. Balinski and H. P. Young, *Fair Representation*, 2nd ed. (Washington: Brookings Institution Press, 2001), p. 31.

[16] For a good survey of these methods, see Laurence F. Schmeckebier, *Congressional Apportionment* (Washington, DC: The Brookings Institution, 1941), pp. 12-58, and *Fair Representation*, pp. 10-35, 60-66.

[17] For a thorough explication of the method of equal proportions, see CRS Report R41357, *The U.S. House of Representatives Apportionment Formula in Theory and Practice*, by (name redacted), esp. pp. 3-7.

Hamilton-Vinton: Ranking Fractional Remainders

Why is there a controversy? Why not apportion the House the intuitive way by dividing each state's population by the national "ideal size" district (710,767 in 2010) and give each state its "quota" (rounding up at fractional remainders of .5 and above, and down for remainders less than .5)? The problem with this proposal is that the House size would not always be 435 seats. In some decades, the House might include 435 seats; in others, it might be either under or over the legal limit. In 2012, this method would have resulted in a 433-seat House (433 in 2002, 438 in 1992).

One solution to this problem of too few or too many seats would be to divide each state's population by the national "ideal" size district, but instead of rounding at the .5 point, allot each state initially the whole number of seats in its quota (except that states entitled to less than one seat would receive one regardless because of constitutional requirements). Next, rank the fractional remainders of the quotas in order from largest to smallest. Finally, assign seats in rank order until 435 are allocated (see **Table 1**). If this system were used in 2012, there would be no difference in the seat distribution relative to the current method (i.e., the method of equal proportions-Hill).

This apportionment formula, which is associated with Alexander Hamilton, was proposed in Congress's first effort to enact an apportionment of the House. The bill was vetoed by President Washington—his first exercise of this power.[18] This procedure, which might be described as the largest fractional remainders method, was used by Congress from 1851 to 1901;[19] but it was never strictly followed because changes were made in the apportionments that were not consistent with the method.[20] It has generally been known as the Vinton method (for Representative Samuel Vinton (Ohio), its chief proponent after the 1850 Census). Assuming a fixed House size, the Hamilton-Vinton method can be described as follows:

> *Hamilton-Vinton*
>
> *Divide the apportionment population[21] by the size of the House to obtain the "ideal congressional district size" to be used as a divisor. Divide each state's population by the ideal district size to obtain its quota. Award each state the whole number obtained in these quotas. (If a state receives less than one Representative, it automatically receives one because of the constitutional requirement.) If the number of Representatives assigned using the whole numbers is less than the House total, rank the fractional remainders of the states' quotas and award seats in rank order from highest to lowest until the House size is reached.[22]*

The Hamilton-Vinton method has simplicity in its favor, but its downfall was the "Alabama paradox." Although the phenomenon had been observed previously, the "paradox" became an issue after the 1880 census when C. W. Seaton, chief clerk of the Census Office, wrote Congress on October 25, 1881, stating,

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18 *Fair Representation*, p. 21. Jefferson's method was used instead for all the apportionments from 1790-1831.

19 Schmeckebier, Congressional Apportionment. p. 73.

20 Fair Representation, p. 37.

21 Currently, the apportionment population is the resident population of the 50 states plus the population of the overseas military and federal employees and their dependents as found by the U.S. Census Bureau.

22 Descriptions of each method of determining an apportionment in this report are blocked and *italicized* to set them off.

> While making these calculations I met with the so-called "Alabama" paradox where Alabama was allotted 8 Representatives out of a total of 299, receiving but 7 when the total became 300.[23]

Alabama's loss of its eighth seat when the House size was increased resulted from the vagaries of fractional remainders. With 299 seats, Alabama's quota was 7.646 seats. It was allocated eight seats based on this quota, but it was on the dividing point. When a House size of 300 was used, Alabama's quota increased to 7.671, but Illinois and Texas now had larger fractional remainders than Alabama. Accordingly, each received an additional seat in the allotment of fractional remainders, but since the House had increased in size by only one seat, Alabama lost the seat it had received in the allotment by fractional remainders for 299 seats.[24] This property of the Hamilton-Vinton method eventually led to a change in the formula in 1911.

One could argue that the "Alabama paradox" should not be an important consideration in apportionments, since the House size was fixed in size at 435, but the Hamilton-Vinton method is subject to other anomalies. Hamilton-Vinton is also subject to the "population paradox" and the "new states paradox."

The population paradox occurs when a state that grows at a greater percentage rate than another has to give up a seat to the slower growing state. The new states paradox works in much the same way—at the next apportionment after a new state enters the Union, any increase in House size caused by the additional seats for the new state may result in seat shifts among states that otherwise would not have happened. Finding a formula that avoided the paradoxes was a goal when Congress adopted a rounding, rather than a ranking, method when the apportionment law was changed in 1911.

Table 1 illustrates how a Hamilton-Vinton apportionment would be done by ranking the fractional remainders of the state's quotas in order from largest to smallest and compares it with simple rounding. In 2011, Minnesota's and Rhode Island's fractional remainders of less than 0.5 would be rounded up by the Hamilton-Vinton method in order for the House to have totaled 435 Representatives.

[23] Fair Representation, p. 38.

[24] Ibid., p. 39.

Table 1. Apportioning the House in 2011 by Simple Rounding and Ranked Fractional Remainders (Hamilton-Vinton)

State	Actual 2010 Apportionment Population	Quota	Seats from Whole Number	Fractional Remainders	Seats from Ordering Higher Fractions	Seat Assignment Using Vinton Method	Seat Assignment Using Hill Method	Difference Between Vinton and Hill Methods	Seat Assignment Using Simple Rounding
North Dakota	675,905	0.951	1	Constitution	0	1	1	0	1
Vermont	630,337	0.887	1	Constitution	0	1	1	0	1
Wyoming	568,300	0.800	1	Constitution	0	1	1	0	1
Tennessee	6,375,431	8.970	8	0.96979	1	9	9	0	9
Michigan	9,911,626	13.945	13	0.94497	1	14	14	0	14
Hawaii	1,366,862	1.923	1	0.92308	1	2	2	0	2
Pennsylvania	12,734,905	17.917	17	0.91713	1	18	18	0	18
New Mexico	2,067,273	2.909	2	0.90851	1	3	3	0	3
Utah	2,770,765	3.898	3	0.89827	1	4	4	0	4
Maine	1,333,074	1.876	1	0.87554	1	2	2	0	2
New Hampshire	1,321,445	1.859	1	0.85918	1	2	2	0	2
Nevada	2,709,432	3.812	3	0.81198	1	4	4	0	4
Alabama	4,802,982	6.757	6	0.75746	1	7	7	0	7
Georgia	9,727,566	13.686	13	0.68601	1	14	14	0	14
West Virginia	1,859,815	2.617	2	0.61663	1	3	3	0	3
Florida	18,900,773	26.592	26	0.59208	1	27	27	0	27
Nebraska	1,831,825	2.577	2	0.57725	1	3	3	0	3
Texas	25,268,418	35.551	35	0.55092	1	36	36	0	36
California	37,341,989	52.538	52	0.53760	1	53	53	0	53
South Carolina	4,645,975	6.537	6	0.53657	1	7	7	0	7
Washington	6,753,369	9.502	9	0.50152	1	10	10	0	10
Rhode Island	1,055,247	1.485	1	0.48466	1	2	2	0	1
Minnesota	5,314,879	7.478	7	0.47767	1	8	8	0	7

State	Actual 2010 Apportionment Population	Quota	Seats from Whole Number	Fractional Remainders	Seats from Ordering Higher Fractions	Seat Assignment Using Vinton Method	Seat Assignment Using Hill Method	Difference Between Vinton and Hill Methods	Seat Assignment Using Simple Rounding
North Carolina	9,565,781	13.458	13	0.45839	0	13	13	0	13
Missouri	6,011,478	8.458	8	0.45773	0	8	8	0	8
Oregon	3,848,606	5.415	5	0.41472	0	5	5	0	5
Louisiana	4,553,962	6.407	6	0.40711	0	6	6	0	6
Montana	994,416	1.399	1	0.39907	0	1	1	0	1
New Jersey	8,807,501	12.392	12	0.39154	0	12	12	0	12
New York	19,421,055	27.324	27	0.32408	0	27	27	0	27
Virginia	8,037,736	11.309	11	0.30854	0	11	11	0	11
Oklahoma	3,764,882	5.297	5	0.29693	0	5	5	0	5
Iowa	3,053,787	4.296	4	0.29647	0	4	4	0	4
Ohio	11,568,495	16.276	16	0.27607	0	16	16	0	16
Delaware	900,877	1.267	1	0.26747	0	1	1	0	1
Massachusetts	6,559,644	9.229	9	0.22897	0	9	9	0	9
Idaho	1,573,499	2.214	2	0.21380	0	2	2	0	2
Mississippi	2,978,240	4.190	4	0.19018	0	4	4	0	4
South Dakota	819,761	1.153	1	0.15335	0	1	1	0	1
Indiana	6,501,582	9.147	9	0.14728	0	9	9	0	9
Maryland	5,789,929	8.146	8	0.14603	0	8	8	0	8
Kentucky	4,350,606	6.121	6	0.12100	0	6	6	0	6
Arkansas	2,926,229	4.117	4	0.11700	0	4	4	0	4
Illinois	12,864,380	18.099	18	0.09929	0	18	18	0	18
Colorado	5,044,930	7.098	7	0.09787	0	7	7	0	7
Connecticut	3,581,628	5.039	5	0.03910	0	5	5	0	5
Kansas	2,863,813	4.029	4	0.02919	0	4	4	0	4
Arizona	6,412,700	9.022	9	0.02223	0	9	9	0	9

State	Actual 2010 Apportionment Population	Quota	Seats from Whole Number	Fractional Remainders	Seats from Ordering Higher Fractions	Seat Assignment Using Vinton Method	Seat Assignment Using Hill Method	Difference Between Vinton and Hill Methods	Seat Assignment Using Simple Rounding
Wisconsin	5,698,230	8.017	8	0.01702	0	8	8	0	8
Alaska	721,523	1.015	1	0.01513	0	1	1	0	1
TOTALS	**309,183,463**		**415**		**20**	**435**	**435**		**433**
Ideal CD Size	710,767								

Source: Data calculated by CRS. The "quota" is found by dividing the state population by the national "ideal congressional district size" (710,767 for 2010). Rhode Island and Minnesota receive additional seats with the Hamilton-Vinton system relative to the method of simple rounding even though their fractional remainders are less than .5.

Rounding Methods

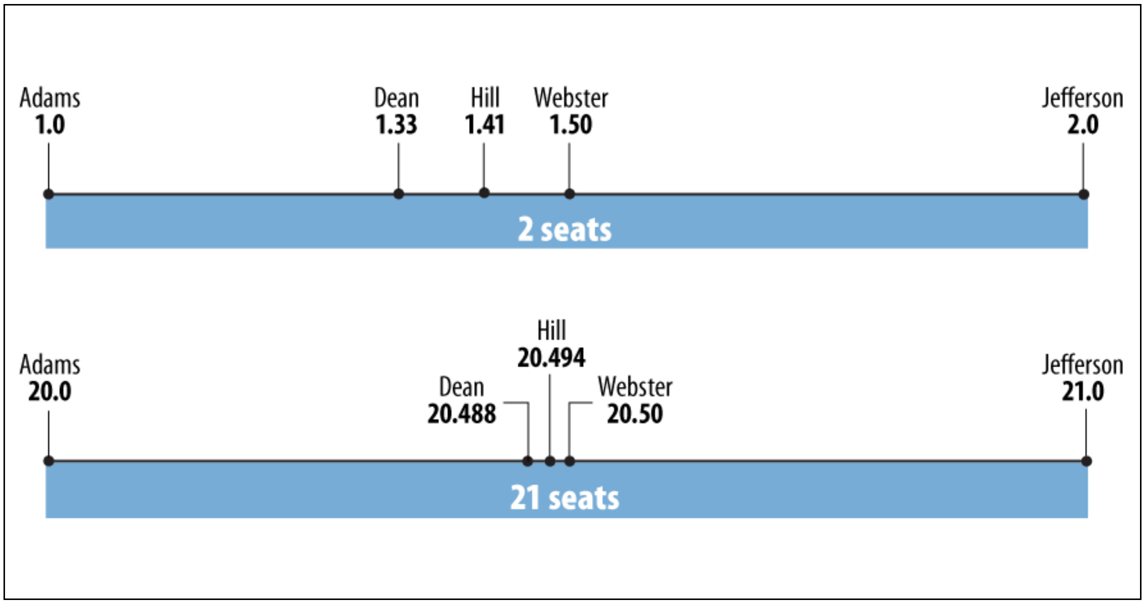
The kinds of calculations required by the Hamilton-Vinton method are paralleled, in their essentials, in all the alternative methods that are most frequently discussed—but fractional remainders are rounded instead of ranked. First, the total apportionment population (the residential population of the 50 states plus the overseas military and federal employees and their dependents as determined by the census) is divided by 435, or the size of the House. This calculation yields the national "ideal" district size. Second, the "ideal" district size is used as a common divisor for the population of each state, yielding what are called the states' quotas of Representatives. Because the quotas still contain fractional remainders, each method then obtains its final apportionment by rounding its allotments either up or down to the nearest whole number according to certain rules.

The operational difference between the methods lies in how each defines the rounding point for the fractional remainders in the allotments—that is, the point at which the fractions rounded down are separated from those rounded up. Each of the rounding methods defines its rounding point in terms of some mathematical quantity. Above this specified figure, all fractional remainders are automatically rounded up; those below are rounded down.

For a given common divisor, therefore, each rounding method yields a set number of seats. If using national "ideal" district size as the common divisor results in 435 seats being allocated, no further adjustment of the divisor is necessary. But if too many or too few seats are apportioned, the common divisor (i.e., the "ideal congressional district size") must be adjusted until a value is found that yields the desired number of seats. (These methods will, as a result, generate allocations before rounding that differ from the states' quotas.) If too many seats are apportioned, a larger divisor is tried (the divisor slides up); if too few, a smaller divisor (it slides down). The divisor finally used is that which apportions a number of seats equal to the desired size of the House.[25]

[25] Balinski and Young, in *Fair Representation*, refer to these as divisor methods because they use a common divisor. This report characterizes them as rounding methods, although they use common divisors, because the Hamilton-Vinton method also uses a common divisor, while its actual apportionment is not based on rounding. All these methods can be described in different ways, but looking at them based on how they treat quotients provides a consistent framework to understand them all.

Figure 1. Illustrative Rounding Points for Five Apportionment Methods (For 2 and 21 Seats)



Source: This illustration is adapted from *Fair Representation*, pp. 63-65.

The rounding methods most often mentioned (although there could be many more) are Webster ("major fractions"); Hill ("equal proportions"—the current method); Dean ("harmonic mean"); Adams ("smallest divisors"); and Jefferson ("greatest divisors"). Under any of these methods, the Census Bureau would construct a priority list of claims to representation in the House.[26] The key difference among these methods is in the rule by which the rounding point is set—that is, the rule that determines what fractional remainders result in a state being rounded up, rather than down.

In the Adams, Webster, and Jefferson methods, the rounding points used are the same for a state of any size. In the Dean and Hill methods, on the other hand, the rounding point varies with the number of seats assigned to the state; it rises as the state's population increases. With these two methods, in other words, smaller (less populous) states will have their apportionments rounded up to yield an extra seat for smaller fractional remainders than will larger states. This property, arguably, provides the intuitive basis for challenging the Dean and Hill methods as favoring small (less populous) states at the expense of the large (more populous) states.[27]

[26] A "priority list" is a list of a state's "right" to the next House seat as determined by the rounding method used. For each seat (51 through 435), each state's "right" to its next seat is calculated. All of these calculations are ranked from highest to lowest and seats are assigned to each state based on the ranking in the "priority list." For a detailed explanation of how apportionments are done using priority lists, as well as how priority lists are constructed, see CRS Report R41357, *The U.S. House of Representatives Apportionment Formula in Theory and Practice*, by (name redacted).

[27] Peyton Young states that the Hill method "systematically favors the small states by 3-4 percent." He determined this figure by first eliminating from the calculations the very small states whose quotas equaled less than one half a Representative. He then computed the relative bias for the methods described in this report for all the censuses based on the "per capita representation in the large states as a group and in the small states as group. The percentage difference between the two is the method's relative bias toward small states in that year. To estimate their long-run behavior, I compute the average bias of each method up to that point in time." See, Brookings Institution Policy Brief No. 88, *Dividing the House: Why Congress Should Reinstate the Old Reapportionment Formula*, p. 4.

These differences among the rounding methods are illustrated in **Figure 1**. The "black dots" in **Figure 1** indicate the points that a state's fractional remainder must exceed for it to receive a second seat, and to receive a 21st seat. **Figure 1** visually illustrates that the only rounding points that change their relative positions are those for Dean and Hill. Using the rounding points for a second seat as the example, the Adams method awards a second seat for any fractional remainder above one. Dean awards the second seat for any fractional remainder above 1.33 (the harmonic mean between 1 and 2). Similarly, Hill gives a second seat for every fraction exceeding 1.41 (the geometric mean between 1 and 2), Webster, 1.5 (the arithmetic mean between 1 and 2), and Jefferson does not give a second seat until its integer value of a state's quotient equals or exceeds two.

Webster: Rounding at the Midpoint (.5)

The easiest rounding method to describe is the Webster ("major fractions") method which allocates seats by rounding up to the next seat when a state has a remainder of .5 and above. In other words, it rounds fractions to the lower or next higher whole number at the arithmetic mean, which is the midpoint between numbers. For example, between 1 and 2 the arithmetic mean is 1.5; between 2 and 3, the arithmetic mean is 2.5, etc. The Webster method (which was used in 1840, 1910, and 1930) can be defined in the following manner for a 435-seat House:

> *Webster*
>
> *Find a number so that when it is divided into each state's population and resulting quotients are rounded at the arithmetic mean, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

Hill: Rounding at the Geometric Mean

The only operational difference between a Webster and a Hill apportionment (equal proportions—the method in use since 1941), is where the rounding occurs. Rather than rounding at the arithmetic mean between the next lower and the next higher whole number, Hill rounds at the geometric mean. The geometric mean is the square root of the multiplication of two numbers. The Hill rounding point between 1 and 2, for example, is 1.41 (the square root of 2), rather than 1.5. The rounding point between 20 and 21 is the square root of 420, or 20.494. The Hill method can be defined in the following manner for a 435-seat House:

> *Hill*
>
> *Find a number so that when it is divided into each state's population and resulting quotients are rounded at the geometric mean, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

Dean: Rounding at the Harmonic Mean

The Dean Method (advocated by Montana in 1991) rounds at a different point—the harmonic mean between consecutive numbers. The harmonic mean is obtained by multiplying the product of two numbers by 2, and then dividing that product by the sum of the two numbers.[28] The Dean rounding point between 1 and 2, for example, is 1.33, rather than 1.5. The rounding point between 20 and 21 is 20.488. The Dean method (which has never been used) can be defined in the following manner for a 435-seat House:

> *Dean*
>
> *Find a number so that when it is divided into each state's population and resulting quotients are rounded at the harmonic mean, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

Adams: All Fractions Rounded Up

The Adams method ("smallest divisors") rounds up to the next seat for any fractional remainder. The rounding point between 1 and 2, for example, would be any fraction exceeding 1 with similar rounding points for all other integers. The Adams method (which has never been used, but was also advocated by Montana) may be defined in the following manner for a 435-seat House:

> *Adams*
>
> *Find a number so that when it is divided into each state's population and resulting quotients that include fractions is rounded up, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

Jefferson: All Fractions Rounded Down

The Jefferson method ("largest divisors") rounds down any fractional remainder. In order to receive 2 seats, for example, a state would need 2 in its quotient, but it would not get 3 seats until it had 3 in its quotient. The Jefferson method (used from 1790 to 1830) can be defined in the following manner for a 435-seat House:

> *Jefferson*
>
> *Find a number so that when it is divided into each state's population and resulting quotients that include fractions is rounded down, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional requirement.)*

Changing the Formula: The Impact in 2011

What would have happened in 2011 if any of the alternative formulas discussed in this report had been adopted? Using the actual 2010 state apportionment population figures provided by the U.S. Census Bureau,[29] **Table 2** and **Table 3**, below display the results of applying the various formulas.

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[28] Expressed as a formula, the harmonic mean (H) of the numbers (A) and (B) is: $H = 2*(A*B)/(A+B)$.

[29] U.S. Census Bureau, Population Division, "2010 Apportionment Results—Apportionment Tables," (continued...)

As compared to the actual apportionment of seats using the Hill method (equal proportions) currently mandated by law, the Dean method, advocated by Montana in 1991, would have resulted (not surprisingly) in Montana regaining its second seat that it lost in 1991, and California losing a seat in 2011. The Webster method would have cost Rhode Island a seat and would have given an additional seat to North Carolina relative to the actual apportionment using the Hill method. The Hamilton-Vinton method would have resulted in a seat distribution that was no different from that produced by using the current (Hill) method. The Adams method in 2011 would have reassigned 18 seats among 15 states (see **Table 2**) relative to the current (Hill) method. The Jefferson method would have reassigned 16 seats among 15 states (see **Table 2**) relative to the current (Hill) method.

Table 2 and **Table 3** below, present seat allocations based on the actual 2010 apportionment population for the six methods discussed in this report. **Table 2** is arranged in alphabetical order. **Table 3** is arranged by total state population, rank-ordered from the most populous state (California) to the least (Wyoming). Allocations that differ from the current method are ***bolded and italicized,*** as well as followed by an ***asterisk*** in both tables.

Table 3 facilitates the evaluation of apportionment methods by looking at their impact according to the population size of the states. For example, it becomes fairly clear from an examination of the beginning and end of the distributions shown in **Table 3** that the Adams method would have taken from the more populous states and would have given to the lesser populated states relative to the current method. On the other hand, it is clear that the Jefferson method would do the opposite relative to the current method.

(...continued)

http://www.census.gov/population/apportionment/data/2010_apportionment_results.html.

Table 2. Seat Assignments in 2010 for Various House Apportionment Formulas

(Alphabetical Order)

State	2010 Apportionment Population	Quota[a]	Smallest Divisors (Adams)	Harmonic Mean (Dean)	Ranked Fractional Remainders (Hamilton-Vinton)	Current Method: Equal Proportions (Hill)	Major Fractions (Webster)	Largest Divisors (Jefferson)
Alabama	4,802,982	6.757	7	7	7	7	7	7
Alaska	721,523	1.015	1	1	1	1	1	1
Arizona	6,412,700	9.022	9	9	9	9	9	9
Arkansas	2,926,229	4.117	4	4	4	4	4	4
California	37,341,989	52.538	***50****	***52****	53	53	53	***55****
Colorado	5,044,930	7.098	7	7	7	7	7	7
Connecticut	3,581,628	5.039	5	5	5	5	5	5
Delaware	900,877	1.267	***2****	1	1	1	1	1
Florida	18,900,773	26.592	***26****	27	27	27	27	27
Georgia	9,727,566	13.686	***13****	14	14	14	14	14
Hawaii	1,366,862	1.923	2	2	2	2	2	2
Idaho	1,573,499	2.214	***3****	2	2	2	2	2
Illinois	12,864,380	18.099	18	18	18	18	18	***19****
Indiana	6,501,582	9.147	9	9	9	9	9	9
Iowa	3,053,787	4.296	***5****	4	4	4	4	4
Kansas	2,863,813	4.029	4	4	4	4	4	4
Kentucky	4,350,606	6.121	6	6	6	6	6	6
Louisiana	4,553,962	6.407	***7****	6	6	6	6	6
Maine	1,333,074	1.876	2	2	2	2	2	***1****
Maryland	5,789,929	8.146	8	8	8	8	8	8
Massachusetts	6,559,644	9.229	9	9	9	9	9	9
Michigan	9,911,626	13.945	14	14	14	14	14	14

State	2010 Apportionment Population	Quota[a]	Smallest Divisors (Adams)	Harmonic Mean (Dean)	Ranked Fractional Remainders (Hamilton-Vinton)	Current Method: Equal Proportions (Hill)	Major Fractions (Webster)	Largest Divisors (Jefferson)
Minnesota	5,314,879	7.478	8	8	8	8	8	**_7*_**
Mississippi	2,978,240	4.190	4	4	4	4	4	4
Missouri	6,011,478	8.458	**_9*_**	8	8	8	8	8
Montana	994,416	1.399	**_2*_**	**_2*_**	1	1	1	1
Nebraska	1,831,825	2.577	3	3	3	3	3	**_2*_**
Nevada	2,709,432	3.812	4	4	4	4	4	4
New Hampshire	1,321,445	1.859	2	2	2	2	2	**_1*_**
New Jersey	8,807,501	12.392	12	12	12	12	12	**_13*_**
New Mexico	2,067,273	2.909	3	3	3	3	3	3
New York	19,421,055	27.324	**_26*_**	27	27	27	27	**_28*_**
North Carolina	9,565,781	13.458	13	13	13	13	**_14*_**	**_14*_**
North Dakota	675,905	0.951	1	1	1	1	1	1
Ohio	11,568,495	16.276	16	16	16	16	16	**_17*_**
Oklahoma	3,764,882	5.297	**_6*_**	5	5	5	5	5
Oregon	3,848,606	5.415	**_6*_**	5	5	5	5	5
Pennsylvania	12,734,905	17.917	**_17*_**	18	18	18	18	18
Rhode Island	1,055,247	1.485	2	2	2	2	**_1*_**	**_1*_**
South Carolina	4,645,975	6.537	7	7	7	7	7	**_6*_**
South Dakota	819,761	1.153	2*	1	1	1	1	1
Tennessee	6,375,431	8.970	9	9	9	9	9	9
Texas	25,268,418	35.551	**_34*_**	36	36	36	36	**_37*_**
Utah	2,770,765	3.898	4	4	4	4	4	4
Vermont	630,337	0.887	1	1	1	1	1	1
Virginia	8,037,736	11.309	11	11	11	11	11	11

State	2010 Apportionment Population	Quota[a]	Smallest Divisors (Adams)	Harmonic Mean (Dean)	Ranked Fractional Remainders (Hamilton-Vinton)	Current Method: Equal Proportions (Hill)	Major Fractions (Webster)	Largest Divisors (Jefferson)
Washington	6,753,369	9.502	10	10	10	10	10	**9***
West Virginia	1,859,815	2.617	3	3	3	3	3	**2***
Wisconsin	5,698,230	8.017	8	8	8	8	8	8
Wyoming	568,300	0.800	1	1	1	1	1	1
TOTALS	**309,183,463**	**435**	**435**	**435**	**435**	**435**	**435**	**435**
Ideal CD Size[b]	710,767							

Source: Congressional Research Service.

Notes:

a. A state's quota of Representatives is obtained by dividing the population of the state by the "Ideal Congressional District (CD) size."

b. The "Ideal CD size" is obtained by dividing the total population of all the states (in this case, 309,183,463) by the size of the House of Representatives, 435.

Table 3. Seat Assignments in 2010 for Various House Apportionment Formulas

(Ranked by State Population)

State	2010 Apportionment Population	Quota[a]	Smallest Divisors (Adams)	Harmonic Mean (Dean)	Ranked Fractional Remainders (Hamilton-Vinton)	Current Method: Equal Proportions (Hill)	Major Fractions (Webster)	Largest Divisors (Jefferson)
California	37,341,989	52.538	***50****	***52****	53	53	53	***55****
Texas	25,268,418	35.551	***34****	36	36	36	36	***37****
New York	19,421,055	27.324	***26****	27	27	27	27	***28****
Florida	18,900,773	26.592	***26****	27	27	27	27	27
Illinois	12,864,380	18.099	18	18	18	18	18	***19****
Pennsylvania	12,734,905	17.917	***17****	18	18	18	18	18
Ohio	11,568,495	16.276	16	16	16	16	16	***17****
Michigan	9,911,626	13.945	14	14	14	14	14	14
Georgia	9,727,566	13.686	***13****	14	14	14	14	14
North Carolina	9,565,781	13.458	13	13	13	13	***14****	***14****
New Jersey	8,807,501	12.392	12	12	12	12	12	***13****
Virginia	8,037,736	11.309	11	11	11	11	11	11
Washington	6,753,369	9.502	10	10	10	10	10	***9****
Massachusetts	6,559,644	9.229	9	9	9	9	9	9
Indiana	6,501,582	9.147	9	9	9	9	9	9
Arizona	6,412,700	9.022	9	9	9	9	9	9
Tennessee	6,375,431	8.970	9	9	9	9	9	9
Missouri	6,011,478	8.458	***9****	8	8	8	8	8
Maryland	5,789,929	8.146	8	8	8	8	8	8
Wisconsin	5,698,230	8.017	8	8	8	8	8	8
Minnesota	5,314,879	7.478	8	8	8	8	8	***7****
Colorado	5,044,930	7.098	7	7	7	7	7	7

State	2010 Apportionment Population	Quota[a]	Smallest Divisors (Adams)	Harmonic Mean (Dean)	Ranked Fractional Remainders (Hamilton-Vinton)	Current Method: Equal Proportions (Hill)	Major Fractions (Webster)	Largest Divisors (Jefferson)
Alabama	4,802,982	6.757	7	7	7	7	7	7
South Carolina	4,645,975	6.537	7	7	7	7	7	***6****
Louisiana	4,553,962	6.407	***7****	6	6	6	6	6
Kentucky	4,350,606	6.121	6	6	6	6	6	6
Oregon	3,848,606	5.415	***6****	5	5	5	5	5
Oklahoma	3,764,882	5.297	***6****	5	5	5	5	5
Connecticut	3,581,628	5.039	5	5	5	5	5	5
Iowa	3,053,787	4.296	***5****	4	4	4	4	4
Mississippi	2,978,240	4.190	4	4	4	4	4	4
Arkansas	2,926,229	4.117	4	4	4	4	4	4
Kansas	2,863,813	4.029	4	4	4	4	4	4
Utah	2,770,765	3.898	4	4	4	4	4	4
Nevada	2,709,432	3.812	4	4	4	4	4	4
New Mexico	2,067,273	2.909	3	3	3	3	3	3
West Virginia	1,859,815	2.617	3	3	3	3	3	***2****
Nebraska	1,831,825	2.577	3	3	3	3	3	***2****
Idaho	1,573,499	2.214	***3****	2	2	2	2	2
Hawaii	1,366,862	1.923	2	2	2	2	2	2
Maine	1,333,074	1.876	2	2	2	2	2	***1****
New Hampshire	1,321,445	1.859	2	2	2	2	2	***1****
Rhode Island	1,055,247	1.485	2	2	2	2	***1****	***1****
Montana	994,416	1.399	***2****	***2****	1	1	1	1
Delaware	900,877	1.267	***2****	1	1	1	1	1
South Dakota	819,761	1.153	***2****	1	1	1	1	1

State	2010 Apportionment Population	Quota[a]	Smallest Divisors (Adams)	Harmonic Mean (Dean)	Ranked Fractional Remainders (Hamilton-Vinton)	Current Method: Equal Proportions (Hill)	Major Fractions (Webster)	Largest Divisors (Jefferson)
Alaska	721,523	1.015	1	1	1	1	1	1
North Dakota	675,905	0.951	1	1	1	1	1	1
Vermont	630,337	0.887	1	1	1	1	1	1
Wyoming	568,300	0.800	1	1	1	1	1	1
TOTALS	**309,183,463**	**435**	**435**	**435**	**435**	**435**	**435**	**435**
Ideal CD Size[b]	710,767							

Source: Congressional Research Service.

Notes:

a. A state's quota of Representatives is obtained by dividing the population of the state by the "Ideal Congressional District (CD) size."

b. The "Ideal CD size" is obtained by dividing the total population of all the states (in this case, 309,142,868) by the size of the House of Representatives, 435.

A Framework for Evaluating Apportionment Methods

All the apportionment methods described above arguably have properties that recommend them. Each is the best formula to satisfy certain mathematical measures of fairness, and the proponents of some of them argue that their favorite meets other goals as well. For example, the major issue raised in the Montana case[30] was, which formula best approximated the "one person, one vote" principle? The apportionment concerns identified in the Massachusetts case[31] not only raised "one person, one vote" issues, but also suggested that the Hill method discriminates against more populous states.

It is not immediately apparent which of the methods described above is the "fairest" or "most equitable" in the sense of meeting the "one person, one vote" standard. As already noted, no apportionment formula can equalize districts precisely, given the constraints of (1) a fixed size House; (2) a minimum seat allocation of one; and (3) the requirement that districts not cross state lines. The practical question to be answered, therefore, is not how inequality can be eliminated, but how it can be minimized. This question too, however, has no clearly definitive answer, for there is no single established criterion by which to determine the equality or fairness of a method of apportionment.

In a report to Congress in 1929, the National Academy of Sciences (NAS) defined a series of possible criteria for comparing how well various apportionment formulas achieve equity among states.[32] This report predates the Supreme Court's enunciation of the "one person, one vote" principle by more than 30 years, but if Congress decided to reevaluate its 1941 choice to adopt the Hill method, it could use one of the NAS criteria of equity as a measure of how well an apportionment formula fulfills that principle.

Although the following are somewhat simplified restatements of the NAS criteria, they succinctly present the question before Congress if it chooses to take up this matter. Which of these measures best approximates the one person, one vote concept?

- The method that minimizes the difference between the largest average district size in the country and the smallest? This criterion leads to the Dean method.
- The method that minimizes the difference in each person's individual share of his or her Representative by subtracting the smallest such share for a state from the largest share? This criterion leads to the Webster method.
- The method that minimizes the difference in average district sizes, or in individual shares of a Representative, when those differences are expressed as percentages? These criteria both lead to the Hill method.

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[30] Department of Commerce v. Montana, 503 U.S. 441 (1992).

[31] *Franklin* v. *Massachusetts,* 505 U.S. 788 (1992).

[32] U.S. Congress, House, Committee on Post Office and Civil Service, Subcommittee on Census and Statistics, *The Decennial Population Census and Congressional Apportionment*, Appendix C: Report of National Academy of Sciences Committee on Apportionment, 91st Cong., 1st Sess., H. Rept. 91-1314 (Washington: GPO, 1970), pp. 19-21.

- The method that minimizes the absolute representational surplus among states?[33] This criterion leads to the Adams method.
- The method that minimizes the absolute representational deficiency among states?[34] This criterion leads to the Jefferson method.

In the absence of further information, it is not apparent which criterion (if any) best encompasses the principle of "one person, one vote." Although the NAS report endorsed as its preferred method of apportionment the one currently in use—the Hill method—the report arguably does not make a clear-cut or conclusive case for one method of apportionment as fairest or most equitable.[35] Are there other factors that might provide additional guidance in making such an evaluation? The remaining sections of this report examine three additional possibilities put forward by statisticians: (1) mathematical tests different from those examined in the NAS report; (2) standards of fairness derived from the concept of states' representational "quotas"; and (3) the principles of the constitutional "great compromise" between large and small states that resulted in the establishment of a bicameral Congress.

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[33] The absolute representational surplus is calculated in the following way. Take the number of Representatives assigned to the state whose average district size is the smallest (the most over-represented state). From this number subtract the number of seats assigned to the state with the largest average district size (the most under-represented state). Multiply this remainder by the population of the most over-represented state divided by the population of the most under-represented state. This number is the absolute representational surplus of the state with the smallest average district size as compared to the state with the largest average district size. In equation form this may be stated as follows: $S= (a-b)*(A/B)$ where S is the absolute representation surplus, A is the population of the over-represented state, B is the population of the under-represented state, a is the number of representatives of the over-represented state, and b is the number of representatives of the under-represented state. For further information about this test, see Schmeckebier, *Congressional Apportionment*, pp. 45-46.

[34] The absolute representational deficiency is calculated in the following way. Take the number of Representatives assigned to the state whose average district size is the largest (the most under-represented state). From this number subtract the number of seats assigned to the state with the largest average district size (the most over-represented state) multiplied by the population of the under-represented state divided by the population of the over-represented state. This number is the absolute representational deficiency of the state with the smallest average district size, as compared to the state with the largest average district size. In equation form, this may be stated as follows: $D=b-((a*B)/A)$ where D is the absolute representation deficiency, A is the population of the over-represented state, B is the population of the under-represented state, a is the number of representatives of the over-represented state, and b is the number of representatives of the under-represented state. For further information about this test, see Schmeckebier, *Congressional Apportionment*, pp. 52-54.

[35] To quote their rationale for selecting the method of equal proportions,

> After full consideration of the various methods, your committee is of the opinion that, on mathematical grounds, the method of equal proportions is the method to be preferred. Each of the other methods is, however, consistent with itself and unambiguous.

The report goes on to state that,

> the best test of a desirable apportionment so far proposed is the following: .....If the "discrepancy" between A/a and B/b (Where A and B represent population and a and b represent number of Representatives) is defined to be the percentage of discrepancy:...

This is accomplished only by using the method of equal proportions. Further, the report states,

> The method of equal proportions is preferred by the committee because it satisfies the test proposed above when applied either to sizes of congressional districts or to numbers of Representatives per person and because it occupies mathematically a neutral position with respect to emphasis on larger or smaller states.

The Decennial Population Census and Congressional Apportionment, p. 21; The last statement of the NAS report was challenged by Balinski and Young in *Fair Representation,* pp. 55,76-78.

Alternative Kinds of Tests

As the discussion of the NAS report showed, the NAS tested each of its criteria for evaluating apportionment methods by its effect on pairs of states. (The descriptions of the NAS tests above stated them in terms of the highest and lowest states for each measure, but, in fact, comparisons between all pairs of states were used.) These pair-wise tests, however, are not the only means by which different methods of apportionment can be tested against various criteria of fairness.

For example, it is indisputable that, as the state of Montana contended in 1992, the Dean method minimizes absolute differences in state average district populations in the pair-wise test. One of the federal government's counter arguments, however, was that the Dean method does not minimize such differences when all states are considered simultaneously. The federal government proposed the variance as a means of testing apportionment formulas against various criteria of fairness.

The variance of a set of numbers is the sum of the squares of the deviations of the individual values from the mean or average.[36] This measure is a useful way of summarizing the degree to which individual values in a list vary from the average (mean) of all the values in the list. High variances indicate that the values vary greatly; low variances mean the values are similar. If all values in the lists are identical, the variance is zero. According to this test, in other words, the smaller the variance is, the more equitable the method of apportionment.

If the variance for a Dean apportionment is compared to that of a Hill apportionment in 2010 (using the difference between district sizes as the criterion), the apportionment variance under Hill's method is smaller than that under Dean's (see **Table 4**). In fact, using average district size as the criterion and variance as the test, the variance under both the Hill method and the Hamilton-Vinton method are the smallest of any of the apportionment methods discussed in this report.

Variances can be calculated, however, not only for differences in average district size, but for each of the criteria of fairness used in pair-wise tests in the 1929 NAS report. As with those pair-wise tests, different apportionment methods are evaluated as most equitable, depending on which measure the variance is calculated. For example, if the criterion used for comparison is the individual share of a Representative, the Webster method proves most effective in minimizing inequality, as measured by variance.

The federal government in the Massachusetts case also presented another argument to challenge the basis for both the Montana and Massachusetts claims that the Hill method is unconstitutional. It contended that percent difference calculations are fairer than absolute differences, because absolute differences are not influenced by whether they are positive or negative in direction.[37]

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[36] In order to calculate variance for average district size, first find the ideal size district for the entire country and then subtract that number from each state's average size district. This may result in a positive or negative number. The square of this number eliminates any negative signs. To find the total variance for a state, multiply this number by the total seats assigned to the state. To find the variance for entire country, sum all the state variances.

[37] Declaration of Lawrence R. Ernst filed on behalf of the Government in Commonwealth *of Massachusetts, et al. v. Mosbacher, et al.* CV NO. 91-111234 (W.D. Mass. 1991), p. 13.

Table 4. Alternate Methods for Measuring Equality of District Sizes

2010 Apportionment Population

	Criteria for Evaluation: Values to Be Minimized			
	Variance		Sum of Absolute Values of Differences	
Method	Average District Size	Individual Shares	Average District Size	Individual Shares
Adams	2,556,870,322	1.78247E-14	14,660,025	3.18873E-05
Dean	1.109,983,360	5.80340E-15	8,634,729	1.77890E-05
Hill (current)	***1,083,482,618***	4.48856E-15	***8,437,775***	1.68894E-05
Webster	1,207,627,381	***3.91151E-15***	8,475,105	***1.65416E-05***
Hamilton-Vinton	***1,083,482,618***	4.48856E-15	***8,437,775***	1.68894E-05
Jefferson	3,539,478,579	7.32662E-15	13,241,999	2.44136E-05

Source: Congressional Research Service

Notes: ***Bolded*** and ***Italicized*** numbers are the smallest for the category. The closer the values are to zero, the closer the method comes to equalizing district sizes in the entire country.

Tests other than pair- wise comparisons and variance can also be applied. For example, **Table 4**, using 2010 Census apportionment population values, reports figures for each method using the sum of the absolute values (rather than the squares) of the differences between national averages and state figures.[38] Using this test for state differences from the national "ideal" both for district sizes and for shares of a Representative, the Hill and Hamilton-Vinton method produce the smallest national totals for deviations from the average district size. The Webster method, on the other hand, minimizes the deviations for shares of a Representative.

Fairness and Quota

These examples, in which different methods best satisfy differing tests of a variety of criteria for evaluation, serve to illustrate further the point made earlier, that no single method of apportionment need be unambiguously the most equitable by all measures. Each apportionment method discussed in this report has a "rational" basis, and for each, there is at least one test according to which it is the most equitable. The question of how the concept of fairness can best be defined, in the context of evaluating an apportionment formula, remains open.

Another approach to this question begins from the observation that, if representation were to be apportioned among the states truly according to population, the fractional remainders would be treated as fractions rather than rounded. Each state would be assigned its exact quota of seats, derived by dividing the national "ideal" size district into the state's apportionment population. There would be no "fractional Representatives," just fractional votes based on the states' quotas.

[38] This is not a "standard" statistical test such as computing the variance. This measure is calculated as follows. Each state's average size district is subtracted from the national "ideal size" district. (In some cases this will result in a negative number, but this calculation uses the "absolute value" of the numbers, which always is expressed as a positive number.) This absolute value for each state is multiplied by the number of seats the method assigns to the state. These state totals of differences from the national ideal size are then summed for the entire nation.

Quota Representation

Congress could weight each Representative's vote to account for how much his or her constituents were either over or under represented in the House. In this way, the states' exact quotas would be represented, but each Representative's vote would count differently. (This might be an easier solution than trying to apportion seats so they crossed state lines, but it would, however, raise other problems relating to potential inequalities of influence among individual Representatives.[39])

If this "quota representation" defines absolute fairness, then the concept of the quota, rather than some statistical test, can be used as the basis of a simple concept for judging the relative fairness of apportionment methods: a method should never make a seat allocation that differs from a state's exact quota by more than one seat.[40] Unfortunately, this concept is complicated in its application by the constitutional requirement that each state must get one seat regardless of population size. Hence, some modification of the quota concept is needed to account for this requirement.

Fair Share

One solution is the concept of "fair share," which accounts for entitlements to less than one seat by eliminating them from the calculation of quota. After all, if the Constitution awards a seat for a fraction of less than one, then, by definition, that is the state's fair share of seats.

> To illustrate, consider a hypothetical country with four states having populations 580, 268, 102, and 50 (thousand) and a House of 10 seats to apportion. Then the quotas are 5.80, 2.68, 1.02 and .50. But if each state is entitled to at least one whole seat, then the fair share of the smallest state is 1 exactly. This leaves 9 seats to be divided among the rest. Their quotas of 9 seats are 5.49, 2.54, and .97. Now the last of these is entitled to 1 seat, so its fair share is 1 exactly, leaving 8 seats for the rest. Their quotas of 8 are 5.47 and 2.53. Since these are both greater than 1, they represent the exact fractional representation that these two states are entitled to; i.e. they are the *fair shares*.[41]

Having accounted for the definitional problem of the constitutional minimum of one seat, the revised measure is not the exact quota, but the states' fair shares. Which method meets the goal of not deviating by more than one seat from a state's fair share? No rounding method meets this test under all circumstances. Of the methods described in this report, only the Hamilton-Vinton method always stays within one seat of a state's fair share. Some rounding methods are better than others in this respect. Both the Adams and Jefferson methods nearly always produce examples of states that get more than one seat above or below their fair shares. Through experimentation we learn that the Dean method tends to violate this concept approximately one percent of the time, while Webster and Hill violate it much less than one percent of the time.[42]

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[39] For example, Virginia's quota of Representatives based on 2010 Census was 11.309. Based on this quota, each Virginia Representative would be entitled to 1.0281 votes each in the House. Their votes would "weigh" more than Wyoming's single Representative whose vote would count 0.800 based on Wyoming's quota.

[40] Fair Representation, p. 79.

[41] Balinski, M. L. and H. P. Young, *Evaluation of Apportionment Methods*, Prepared Under a Contract for the Congressional Research Service of the Library of Congress, Contract No. CRS 84-15, September 30, 1984, p. 3. Available to congressional staff from the author upon request.

[42] Ibid., p. 16.

Implementing the "Great Compromise"

The framers of the Constitution (as noted earlier) created a bicameral Congress in which representation for the states was equal in the Senate and apportioned by population in the House. In the House, the principal means of apportionment is by population, but each state is entitled to one Representative regardless of its population level. Given historians' understanding that the "great compromise" was struck, in part, in order to balance the interests of the smaller states with those of the larger ones, how well do the various methods of apportionment contribute to this end?

If it is posited that the combination of factors favoring the influence of small states encompassed in the great compromise (equal representation in the Senate, and a one seat minimum in the House) unduly advantages the small states, then compensatory influence could be provided to the large states in an apportionment formula. This approach would suggest the adoption of the Jefferson method because it significantly favors large states.[43]

If it is posited that the influence of the small states is overshadowed by the larger ones (perhaps because the dynamics of the electoral college focus the attention of presidential candidates on larger states, or the increasing number of one-Representative states—from five to seven since 1910), there are several methods that could reduce the perceived imbalance. The Adams method favors small states in the extreme, Dean much less so, and Hill to a small degree.[44]

If it is posited that an apportionment method should be neutral in its application to the states, two methods may meet this requirement. Both the Webster and Hamilton-Vinton methods are considered to have these properties.[45]

Summary and Overview

If Congress decides to revisit the matter of the apportionment formula, this report illustrates that there could be many alternative criteria from which it can choose as a basis for decision. Among the competing mathematical tests are the pair-wise measures proposed by the National Academy of Sciences in 1929. The federal government proposed the statistical test of variance as an appropriate means of computing a total for all the districts in the country in the 1992 litigation on this matter. The plaintiffs in Massachusetts argued that the variance can be computed for different criteria than those proposed by the federal government—with different variance measures leading to different methods.

The contention that one method or another best implements the "great compromise" is open to much discussion. All of the competing points suggest that Congress would be faced with difficult choices if it decided to take this issue up prior to the 2020 Census. Which of the mathematical

[43] **Table 3** displays the rank-orders of the states by their projected 2010 populations. The Jefferson method awards 55 seats to California and 37 seats to Texas when these states' quotas (state population divided by 1/435 of the apportionment population) are 52.42 and 35.32 respectively.

[44] There is disagreement on this point as it pertains to the Hill method (*Declaration of Lawrence R. Ernst*) but the evidence that the Hill method is slightly biased toward small states is more persuasive than the criticism. See Balinski and Young, *Evaluation of Apportionment Methods*, noted above.

[45] Evaluation of Apportionment Methods, pp. 10-12.

tests discussed in this report best approximates the constitutional requirement that Representatives be apportioned among the states according to their respective number is, arguably, a matter of judgment—rather than an indisputable mathematical test.

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